Behaviour of Elastic Waves
In a Fluid-Saturated Porous Medium
with a Structure

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Abstract. In this paper the results of the study of the influence of a porous medium structure on the propagation of waves in a fluid-saturated porous medium are presented. The medium consists of an isotropic elastic solid skeleton with pores filled by viscous compressible fluid. The isotropic pore structure is characterized by two parameters: the volume porosity and the structural permeability parameter. The first problem studied in this paper concerns the waves produced by the harmonic forces applied to the boundary of a fluid-saturated half-space. As usually in such a medium there are propagated two longitudinal waves: the wave of the first kind (fast wave) and the wave of the second kind (slow wave). The second problem is connected with the study of surface wave propagation in the considered medium. For lower frequencies the surface wave is not sensitive on the medium structure. For higher frequencies the phase velocity of the wave increases with increasing structure parameter however the measure of dissipation decreases with increasing structure parameter.

INTRODUCTION

The aim of this paper is to present the results of the study of the influence of a porous medium structure on the propagation of waves in the fluid-saturated porous medium. The medium consists of an isotropic elastic solid skeleton with pores filled by viscous compressible fluid. Usually in the soil and rock mechanics the structure of a porous medium is determined by one, the well known parameter, volume porosity \( f_v \). However such description of a porous medium structure is too poor to describe the dynamic properties of this medium from the point of view of balance of momentum leading to the equations of motion. This is a reason of introducing so called double-parameter description of a porous medium structure. Complete description of a porous medium by two parameters has been given by Kubik [3]. This author characterized the isotropic pore structure by two parameters: the volume porosity \( f_v \) and the structural permeability parameter \( \lambda \) or structure parameter \( \kappa = \lambda / f_v \).
The first problem presented in this paper concerns the waves produced by the harmonic forces applied to the boundary of a fluid-saturated half-space. The second problem is connected with the study of the surface wave propagation in the considered medium.

**SYSTEMS OF WAVE EQUATIONS**

We confine our considerations to the elastic fluid-saturated porous medium described by the linear constitutive relations proposed by Biot, [8],

\[
T^s = 2NE^s + (AtrE^s + QtrE^f)1, \quad (1)
\]

\[
T^f = (QtrE^s + RtrE^f)1, \quad (2)
\]

where \(A, N, Q, R\) are the material constants of a medium, \(E^s\) and \(E^f\) are the linear strain tensors. The wave equations for the considered medium are of the form

\[
P \nabla^2 \varphi_s + Q \nabla^2 \varphi_f = \rho_{11}(\kappa) \frac{\partial^2 \varphi_s}{\partial t^2} + \rho_{12}(\kappa) \frac{\partial^2 \varphi_f}{\partial t^2} + b \left( \frac{\partial \varphi_s}{\partial t} - \frac{\partial \varphi_f}{\partial t} \right),
\]

\[
Q \nabla^2 \varphi_s + R \nabla^2 \varphi_f = \rho_{12}(\kappa) \frac{\partial^2 \varphi_s}{\partial t^2} + \rho_{22}(\kappa) \frac{\partial^2 \varphi_f}{\partial t^2} - b \left( \frac{\partial \varphi_s}{\partial t} - \frac{\partial \varphi_f}{\partial t} \right),
\]

and

\[
N \nabla^2 h = \rho_{11}(\kappa) \frac{\partial^2 h}{\partial t^2} + \rho_{12}(\kappa) \frac{\partial^2 g}{\partial t^2} + b \left( \frac{\partial h}{\partial t} - \frac{\partial g}{\partial t} \right),
\]

\[
0 = \rho_{12}(\kappa) \frac{\partial^2 h}{\partial t^2} + \rho_{22}(\kappa) \frac{\partial^2 g}{\partial t^2} - b \left( \frac{\partial h}{\partial t} - \frac{\partial g}{\partial t} \right).
\]

Where

\[
b = \mu \frac{f^2}{k_p}, \quad (5)
\]

\[
\rho_{11}(\kappa) = \bar{\rho}^s + \frac{1-\kappa}{\kappa} \bar{\rho}^f, \quad \rho_{12}(\kappa) = -\frac{1-\kappa}{\kappa} \bar{\rho}^f, \quad \rho_{22}(\kappa) = \frac{1}{\kappa} \bar{\rho}^f, \quad (6)
\]

\(\mu\) is the dynamic viscosity of the fluid and \(k_p\) is the Darcy’s coefficient of permeability. The mass coefficients \(\rho_{11}(\kappa), \rho_{12}(\kappa), \rho_{22}(\kappa)\) depend on the structure parameter \(\kappa\). Thus, the influence of a medium structure on wave propagation stems from these terms of the displacement equations. Two systems of wave equations (3) and (4) constitute the point of departure to our considerations.
WAVES IN A FLUID-SATURATED POROUS HALF-SPACE

The first problem studied in this paper concerns the wave produced by the harmonic uniformly distributed load applied to the boundary of a fluid-saturated porous half-space. The dynamical load of the boundary of half-space causes, as usually in such a medium, the propagation of two longitudinal waves: the wave of the first kind (fast wave) and the wave of the second kind (slow wave). The solution of the dispersion equation gives us the phase velocities and attenuation coefficients both of the waves as functions of angular frequency of the wave and structure parameter. In order to estimate the influence of a medium structure on the propagation of longitudinal waves due to the uniformly distributed load of the boundary of porous half-space medium the numerical calculations were performed taking the data given by Fatt [2] for oil-saturated sandstone and by Yew and Jogi [4] for water-saturated Bere’a sandstone: The results of the numerical calculations are shown in figures.

Fig. 1. Dimensionless phase velocities of the waves of the first kind and the wave of the second kind versus dimensionless angular frequency for lower frequencies

Fig. 2. Dimensionless attenuation coefficients of the waves of the first kind and the wave of the second kind versus dimensionless angular frequency for lower frequencies
The second problem is connected with the study of surface waves propagation in a fluid-saturated porous half-space with a structure. This problem is of interest in earthquake engineering applications. To estimate the influence of a medium structure on the propagation of a surface wave the numerical calculations were performed taking the data for oil-saturated sandstone.

CONCLUSIONS

The obtained results indicate that the influence of a medium structure on the phase velocities of the longitudinal waves increases with the increase of frequency of the wave. Generally the phase velocities of the waves are greater in the medium with a structure than in the fluid-saturated porous medium described by volume porosity only. The influence of a medium structure on the phase velocities of the longitudinal waves increases with the increase of frequency of the wave. Structure of the medium causes stronger dispersion. The attenuation of the wave of the first kind depend much stronger on the frequency of the wave than the attenuation of the wave of the second kind. The attenuation both of the waves is very sensitive to structure of a medium. In the medium with structure the attenuation of the wave is greater than in the medium described by volume porosity only.

For lower frequencies the surface wave is not sensitive on the medium structure. This conclusion concerns especially the frequencies observed in seismology. For higher frequencies the phase velocity of the wave increases with increasing structure parameter however the measure of dissipation decreases with increasing structure parameter. In the medium with structure the phase velocity of the wave is greater and the measure of dissipation is lower than in the medium described by one parameter only.

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REFERENCES
