Waves in saturated poro-elastic media with gas bubbles.

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Abstract. It is found that in such a system at gas bubble resonance the reconstruction of an oscillation spectrum takes place: three waves appear instead of the two Frenkel - Biot waves. These three waves include two acoustic modes and the third wave is "optical" if to use conventional terminology. The latter has a frequency higher than the resonant one. In other words, the second wave with its essential matrix deformation has two own modes due to the bubble presence in a pore space. Matrix volume changes happen only if practically incompressible fluid has to leave a pore under its compression or the fluid has to intrude into the bubble space. After resonance these two ways are realized both (before only the first one). At the same frequency the first P-wave becomes slow - due to compressibility growth - and gets high attenuation dependent only on bubble dynamics but not on fluid viscosity. If saturating fluid is not viscous, the 2^{nd} wave is amplified in this frequency range. These spectrum features are kept even at very small gas bubble presence (fractions of percent). As the result, before the resonance the 1^{st} wave is observed *in situ*, after – the 2^{nd} one.

INTRODUCTION

The problem of the Biot 2^{nd} (slow) P-wave [1] is being discussed up to now. If it is the only observable seismic wave [2] in “dry” or practically “dry” soils and rocks, running with low velocity, under full liquid saturation, the 1^{st} (fast) P-wave becomes the visible seismic wave but the 2^{nd} can spread at extremely short distances [3].

It is quite reasonable to think that at intermediate values of gas presence, we could see that seismic waves change their types. The alternative could be that both P-waves exist in reality [2]. Correspondingly, in some papers some data were published that could be interpreted as transitions from one wave type to another one or that there are more than one P-wave running in the case of partial saturation media [3].

The problem is resolved here by numerical calculations of the dynamic poro-mechanics equation in form [4, 5] but with added dynamics of gas bubbles in a fluid phase.

DYNAMIC EQUATIONS

Plane one-dimensional waves (along axis z) are described by the following mass and impulse balances for two phases (symbols 1 and 2), see [4,5]

$$\frac{\partial(1-m)\rho_1}{\partial t}+\frac{\partial(1-m)\rho_1 v_1}{\partial z}=0 \tag{1}$$

$$\frac{\partial m\rho_2}{\partial t}+\frac{\partial m\rho_2 v_2}{\partial z}=0 \tag{2}$$

$$(1-m)\rho_1\frac{dv_1}{dt}=-(1-m)\frac{dP}{dz}+\frac{\partial\sigma^f}{\partial z}+\frac{m^2\mu(v_2-v_1)}{k} \tag{3}$$

$$\rho_2 m\frac{dv_2}{dt}=-m\frac{\partial P}{\partial z}-\frac{m^2\mu(v_2-v_1)}{k}. \tag{4}$$

Because of gas bubble presence, the saturating fluid is described by two differential equations – for number n of bubbles in a unit volume

$$\frac{\partial n}{\partial t}+\frac{\partial n v_2}{\partial z}=0 \tag{5}$$

and for bubble dynamics (of the Lamb – Raleigh type) of radius a:

$$a\frac{d^2a}{dt^2}+\frac{3}{2}\left(\frac{da}{dt}\right)^2+\frac{4\mu_2}{a\rho_2}\left(1+\frac{ma^2}{4\psi k}\right)\frac{da}{dt}=\frac{P_g(a)-P}{\rho_l} \tag{6}$$

Here velocities v_1, v_2 and densities ρ_1, ρ_2, of phases; pore pressure P, σ^f – effective stress in the matrix; m – porosity; k – permeability; μ_2 - fluid viscosity; P_g – gas pressure inside a bubble; ρ_l – liquid density: $\rho_2 = \rho_l(1-\psi)$; $\psi=(4\pi/3)a^3 n$ - gas saturation of the fluid phase. We neglect changes of viscosity and thermal effects.

Equations (1)-(6) are completed with the constitutive laws [4,5]:

$$\sigma^f=(K+\frac{4}{3}G)e+\varepsilon P \tag{7}$$

$$\frac{\rho_1}{\rho_{10}}-1=\beta\left(P-P_0-\frac{K}{1-m}e\right);\ \ \rho_l-\rho_{l0}=\frac{P-P_0}{c_l^2};\ \ P_g=P_0\left(\frac{a_0}{a}\right)^{3\gamma} \tag{8}$$

Here $\varepsilon=\beta K$ and K, β^{-1} – modulus of volume elasticity of the matrix and its intact material, G – matrix shear modulus, e – matrix strain. It is assumed that pore pressure P is equal to liquid pressure far from a bubble centre. The index “0” corresponds to equilibrium state.

NUMERICAL STUDY OF DISPERSION

For this aim we use the linear variant of equations (1)-(8) and look for waves:

$$\mathbf{Y} = \mathbf{Y_0} \exp i(\omega t - \eta z) \qquad (9)$$

Here ω is a frequency and η is a wave number.

The wave velocity and the amplification ($\Delta>0$)/attenuation ($\Delta<0$) are determined as: $V(\eta)=\omega/\mathrm{Re}(\eta)$ and $\Delta(\eta)=\mathrm{Im}(\eta)$. $c_m^2 = [K+(4/3)G]/(1-m_0)\rho_1$; ω_r is resonance frequency of bubbles.

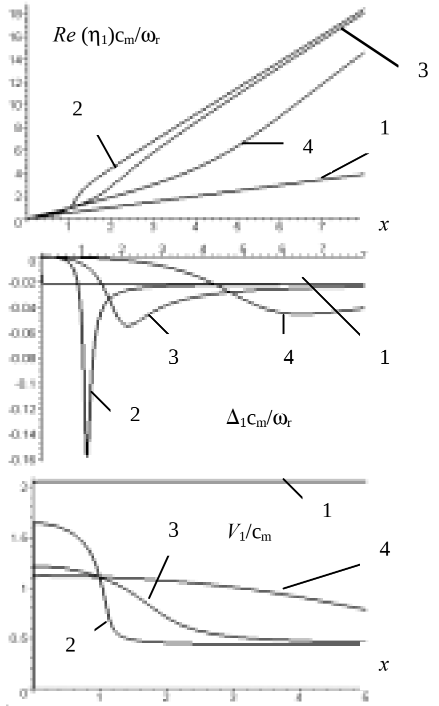

Figure 1. Dependence of wave number *Re* $(\eta_1)c_m/\omega_r$; attenuation coefficient $\Delta c_m/\omega_r$; wave velocity V_1/c_m on frequency $x = \omega/\omega_r$ of the 1st wave with dissipation. 1.Ψ = 0%; 2. Ψ = 0.01%; 3. Ψ = 0.1%; 4. Ψ = 1%.

We use MAPLE-6 for numerical experiment. The numerical results are given for the cases of the 1st P-wave with dissipation (μ=1 cp) in Fig. 1. It is found that wave number spectrum and velocity dispersion is independent on the viscosity. This means that the bubbles effect is absolutely predominant.

The 1st wave spectrum inclination is changing essentially at the bubble resonance. The wave velocity at high frequency became even less than velocity of the 2nd wave. The maximum dissipation of the 1st wave at the resonant frequency (Fig.1, middle plot).

At a small amount of gas bubbles ($\psi\neq0$), the conventional Biot 2nd wave spectrum is changed principally. In the first turn let consider the case of the absence of dissipation (Fig. 2). As it can be seen, there are two branches of the 2nd wave, separated at the bubble resonant frequency ω_r. At $\omega<\omega_r$ there is the Biot acoustical branch, but at $\omega>\omega_g$ the high frequency "optical" oscillations are happened in the running 2nd P-wave (here ω_g – some boundary frequency).

In the gap $\omega_r<\omega<\omega_g$, square of wave number is less than zero (we have the interval of wave non-transparency). The account of the viscosity suppresses the latter effect in the 2nd wave (Fig. 3): "optical" wave appears. However, in the former gap zone wave attenuation and wave velocity are both extremely high. Only at higher frequency the 2nd wave will have a low attenuation and its wave velocity becomes higher than the 1st wave velocity.

As it follows from these results, in situ the 1st wave can be observed at low frequencies and the new "optical" mode of the 2nd wave at high frequencies.

More details of these calculations will be available in [6].

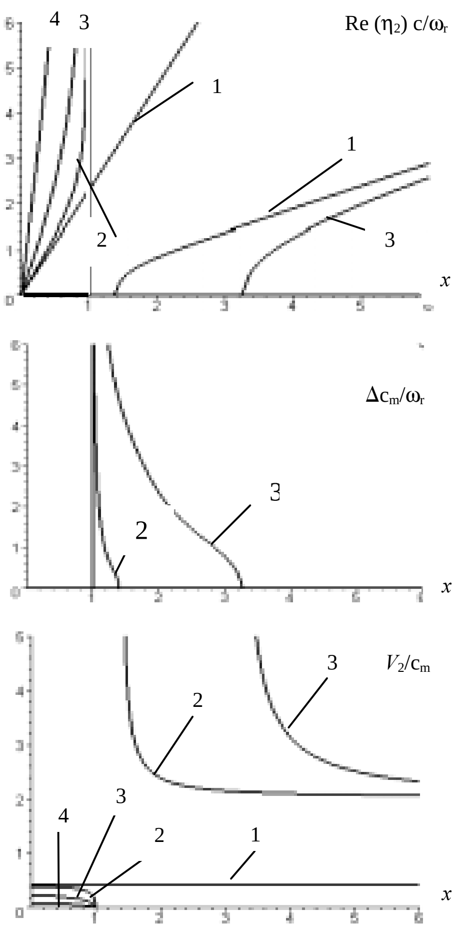

Figure 2. The 2^{st} wave without dissipation.

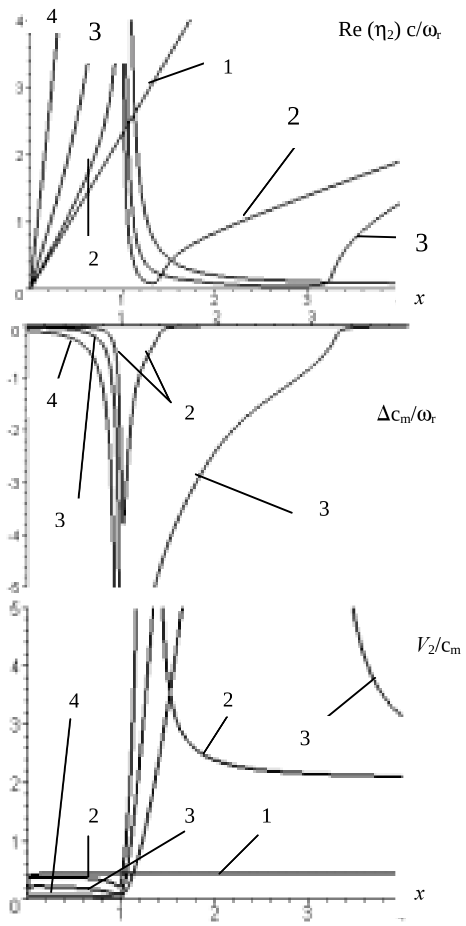

Figure 3. The 2^{st} wave with dissipation.

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